PHYSICS EQUATIONS & ANSWERS™

Essential Tool for Physics Laws, Concepts, Variables & Equations, Including ✎ Sample Problems, ⚠ Common Pitfalls & ⌕ Helpful Hints

BASICS

A. Units for Physical Quantities

Base Units	Symbol	Unit
Length	l, x	Meter - m
Mass	m, M	Kilogram - kg
Temperature	T	Kelvin - K
Time	t	Second - s
Electric Current	I	Ampere - A (C/s)
Derived Units	**Symbol**	**Unit**
Acceleration	a	m/s^2
Ang. Accel.	α	radian/s^2
Ang. Momentum	L	kg m^2/s
Ang. Velocity	ω	radian/sec
Angle	θ, φ	radian
Capacitance	C	Farad F (C/V)
Charge	Q, q, e	Coulomb C (A s)
Density	ρ	kg/m^3
Displacement	s, d, h	meter - m
Electric Field	E	V/m
Electric Flux	Φ_e	V m
Electromotive Force (EMF)	E	Volt - V
Energy	E, U, K	Joule J (kg m^2 s^{-2})
Entropy	S	J/K
Force	F	Newton - N (kg m/s^2 = J/m)
Frequency	f, ν	Hertz - Hz (cycle/s)
Heat	Q	Joule - J
Magnetic Field	B	Tesla (Wb/m^2)
Magnetic Flux	Φ_m	Weber Wb (kg m^2/A s^2)
Momentum	p	kg m/s
Potential	V	Voltage V (J/C)
Power	P, P	Watt - W (J/s)
Pressure	P	Pascal - Pa (N/m^2)
Resistance	R	Ohm Ω (V/A)
Torque	τ	N m
Velocity	v	m/s
Volume	V	m^3
Wavelength	λ	meter - m
Work	W	Joule - J (N m)

B. Fundamental Physical Constants

Base Units	Symbol	Unit
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Avogadro Constant	N_A	6.022×10^{23} mol^{-1}
Elementary charge	e	1.602×10^{-19} C
Faraday Constant	F	96,485 C/mol
Speed of light	c	3×10^8 m s^{-1}
Molar Gas Constant	R	8.314 J mol^{-1} K^{-1}
Boltzmann Constant	k	1.38×10^{-23} J K^{-1}
Gravitation Constant	G	6.67×10^{-11} m^3 kg^{-1} s^{-2}
Permeability of Space	μ_0	$4\pi\times10^{-7}$ N/A^2
Permittivity of Space	ε_0	8.85×10^{-12} F/m

C. Conversion factors and alternative units

	Unit	Description
Angle	° (degree)	180° = π rad
Energy	Erg	CGS unit (g cm^2/s^2) 1 erg = 10^{-7} J
Energy	Electron Volt	1 eV = 1.602×10^{-19} J
Force	Dyne	CGS unit (g cm/s^2 = erg/cm) 1 dyne = 10^{-5} N
Volume	Liter	1 L = 1 dm^3
Pressure	Bar	1 Bar = 10^5 Pa
Length	Angstrom	1 Å = 1×10^{-10}m

MATHEMATICAL CONCEPTS

A. Vector Algebra

1. **Vector**: Denotes directional character using (x, y, z) components fig 1
   a. **Unit vectors**: **i** along x, **j** along y, **k** along z
   b. **Vector** $\boldsymbol{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$
   c. **Length of** $\boldsymbol{A} = |\boldsymbol{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$
2. **Addition of vectors** $\boldsymbol{A}$ **&** $\boldsymbol{B}$, add components:
   $\boldsymbol{A} + \boldsymbol{B} = (A_x+B_x)\mathbf{i} + (A_y+B_y)\mathbf{j} + (A_z+B_z)\mathbf{k}$
   ✎ **Sample Addition and Length Calculations**:

$\boldsymbol{A}=3\mathbf{i}+4\mathbf{j}-3\mathbf{k}$ $\quad |\boldsymbol{A}| = \sqrt{9+16+9} = \sqrt{34} = 5.83$

$\boldsymbol{B}=-2\mathbf{i}+6\mathbf{j}+5\mathbf{k}$ $\quad |\boldsymbol{B}| = \sqrt{4+36+25} = \sqrt{65} = 8.06$

$\boldsymbol{A}+\boldsymbol{B}=\mathbf{i}+10\mathbf{j}+2\mathbf{k}$ $\quad |\boldsymbol{A}+\boldsymbol{B}| = \sqrt{1+100+4} = \sqrt{105} = 10.25$

Note: $|\boldsymbol{A}| + |\boldsymbol{B}| \geq |\boldsymbol{A} + \boldsymbol{B}|$

3. **Multiply** $\boldsymbol{A}$ **&** $\boldsymbol{B}$:
   a. Dot or **scalar product**: $\boldsymbol{A} \cdot \boldsymbol{B} = |\boldsymbol{A}|\,|\boldsymbol{B}|\cos\theta = (A_xB_x) + (A_yB_y) + (A_zB_z)$
   Note: θ is the angle between $\boldsymbol{A}$ and $\boldsymbol{B}$;
   $\boldsymbol{A} \cdot \boldsymbol{B} = 0$, if $\theta = \pi/2$ fig 2
   ✎ **Sample: Scalar product**:
   $\boldsymbol{A} = 5\mathbf{i} + 2\mathbf{j}$ $\quad \boldsymbol{B} = 3\mathbf{i} + 5\mathbf{j}$
   $\boldsymbol{A} \cdot \boldsymbol{B} = 3\times5 + 2\times5 = 15 + 10 = 25$
   $|\boldsymbol{A}| = \sqrt{25+4} = \sqrt{29} = 5.385$
   $|\boldsymbol{B}| = \sqrt{9+25} = \sqrt{34} = 5.831$
   $\cos\theta = \dfrac{\boldsymbol{A}\cdot\boldsymbol{B}}{|\boldsymbol{A}||\boldsymbol{B}|} = \dfrac{25}{5.385\times5.83} = 0.796$
   $\theta = \cos^{-1}(0.796) = 37° = 0.2\pi$ rad fig 3

   b. Cross or **Vector Product**:
   $\boldsymbol{C} = \boldsymbol{A}\times\boldsymbol{B} = |\boldsymbol{A}|\,|\boldsymbol{B}|\sin\theta\,\mathbf{e}$
   θ – Angle between $\boldsymbol{A}$ and $\boldsymbol{B}$, vector **e** is perpendicular to $\boldsymbol{A}$ and $\boldsymbol{B}$

$$\boldsymbol{A}\times\boldsymbol{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

✎ **Sample: Vector Product**:
$\boldsymbol{A} = 2\mathbf{i} + \mathbf{j}$ $\quad \boldsymbol{B} = \mathbf{i} + 3\mathbf{j}$

$$\boldsymbol{A}\times\boldsymbol{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 0 \\ 1 & 3 & 0 \end{vmatrix} = (6-1)\,\mathbf{k} = 5\mathbf{k}$$

- If $\boldsymbol{A}$ and $\boldsymbol{B}$ are in x-y plane, $\boldsymbol{A}\times\boldsymbol{B}$ is along the + z direction
- θ is the angle formed by $\boldsymbol{AB}$: $\sin\theta = \dfrac{|C|}{|A||B|}$

Given: $|\boldsymbol{A}| = \sqrt{5}$ $\quad |\boldsymbol{B}| = \sqrt{10}$ $\quad |\boldsymbol{C}| = 5$
$\sin\theta = 5/(\sqrt{5}\times\sqrt{10}) = 5/\sqrt{50} = 1/\sqrt{2}$
$< \boldsymbol{AB}$: $\theta = 45° = \pi/2$ radians

   c. The **Right-Hand Rule** gives the orientation of vector **e** fig 4

B. Trigonometry

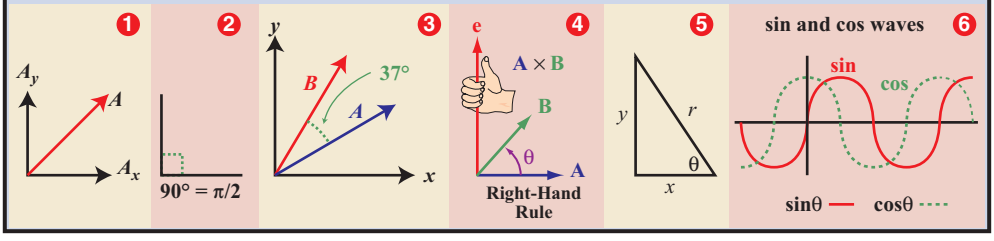
1. **Basic relations for a triangle** fig 5

$\sin\theta = \dfrac{y}{r}$ $\quad \cos\theta = \dfrac{x}{r}$ $\quad \tan\theta = \dfrac{y}{x} = \dfrac{\sin}{\cos}$ $\quad \sin^2 + \cos^2 = 1$

Values Of sin, cos and tan

θ rad [°]	sin θ	cos θ	tan θ
0 [0°]	0.00	1.00	0.00
π/6 [30°]	0.50	0.866	0.577
π/4 [45°]	0.707	0.707	1.00
π/3 [60°]	0.866	0.50	1.732
π/2 [90°]	1.00	0.00	∞
π [180°]	0.00	−1.00	0.00

2. **Sinand Cos waves** fig 6



MATHEMATICAL CONCEPTS (cont.)

C. **Geometry** 7

Circle: Area = πr^2; Circumference = $2\pi r$

Sphere: Volume = $^4/_3\pi r^3$; Area = $4\pi r^2$

Cylinder: Volume = $h\pi r^2$

Triangle: Sum of angles = 180° fig 7

D. **Coordinate Systems**

1. **One dimension (1-D)**: position = x fig 8
   - The x position is described relative to an origin

2. **Two dimensions (2-D)** fig 9

$x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2 + y^2$

a. Calculate (r, θ) from (x, y):

$r = \sqrt{x^2 + y^2}$; $\theta = \sin^{-1}\left(\frac{y}{r}\right)$

b. Calculate (x, y) from (r, θ), or x and y components of a vector "r" with angle θ;

$x = r\cos\theta$; $y = r\sin\theta$

✎ **Sample**: Generate x and y vector components, given: $r = 5.0$, $\theta = \frac{\pi}{6}$ (30°)

$x = r\cos\left(\frac{\pi}{6}\right) = 5.0 \times 0.866 = 4.33$

$y = r\sin\left(\frac{\pi}{6}\right) = 5 \times 0.50 = 2.50$

Check your work: $x^2 + y^2 = r^2$

$2.5^2 + 4.33^2 = 6.25 + 18.75 = 25.00$

It checks, $r^2 = 25.00$

3. **Three Dimensions (3-D)**

a. **Cartesian (*x*, *y*, *z*)**: The basic coordinate system

b. **Cylindrical**: (r, θ, z) fig 10
   - Polar coordinates, with a z axis
   - Calculate (r, θ) from (x, y); calculate (x, y) from (r, θ)
   - Same process as for 2-d polar; z: same as Cartesian

c. **Spherical**: (r, θ, φ)

$x = r\ \sin\varphi\cos\theta$, $y = r\ \sin\varphi\sin\theta$,

$z = r\cos\varphi$, $r^2 = x^2 + y^2 + z^2$ fig 11
   - Calculate (r, θ, φ) from (x, y, z)
   - Calculate (x, y, z) from (r, θ, φ)

🔎 **Hint:** Follow the strategy for 2-d polar coordinates

E. Use of **Calculus** in Physics

1. Methods from calculus are used in physics definitions, and the derivations of equations and laws

Physical meanings of calculus expressions:

a. Derivative - slope of the curve: $\frac{dF(x)}{dx}$

b. Integral - area under the curve: $\int F(x)dx$

✎ **Samples**:
   - Position: x or $F(x)$

Velocity: $v(x) = \frac{dF(x)}{dt}$

Acceleration: $a = \frac{dv(x)}{dt}$
   - Power and work: $P = \frac{dW}{dt}$
   - Energy and force: $E = \int F dx$

2. Other useful expressions:

a. $\frac{d(F \cdot G)}{dx} = F\frac{dG}{dx} + G\frac{dF}{dx}$

b. $\frac{d(F \div G)}{dx} = \frac{1}{G}\frac{dF}{dx} - \frac{F}{G^2}\frac{dG}{dx}$

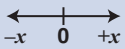
c. **Partial derivative**: $\frac{\partial F(x,y,z)}{\partial x} = \frac{dF}{dx}$, hold y & z constant

d. **Gradient Operator ∇ (Del)** = $\partial/\partial x + \partial/\partial y + \partial/\partial z$

e. **Integration by parts**: $\int u dv = uv - \int v du$

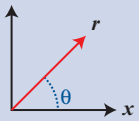
f. Symbol for integration of closed surface or volume: $\oint$

8



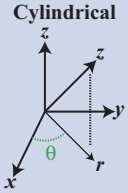
$v = dx/dt$;
$a = dv/dt$

9 **Polar**



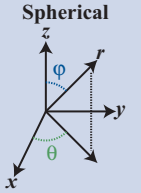
Cartesian: (x, y)
Polar: (r, θ)

10 **Cylindrical**



$x = r\cos\theta$,
$y = r\sin\theta$,
$r^2 = x^2 + y^2$

11 **Spherical**



$x = r\ \sin\varphi\cos\theta$,
$y = r\ \sin\varphi\sin\theta$,
$z = r\cos\varphi$,
$r^2 = x^2 + y^2 + z^2$

Common derivatives and integrals

$F(x)$	$\frac{dF(x)}{dx}$	$\int F(x)\,dx$
constant	0	constant x
x	1	$\frac{1}{2}x^2$
x^2	$2x$	$\frac{1}{3}x^3$
x^n	nx^{n-1}	$\frac{1}{n+1}x^{n+1}$
$\frac{1}{x}$	$\frac{-1}{x^2}$	$\ln x$
$\ln x$	$\frac{1}{x}$	$x \ln x - x$
e^x	e^x	e^x
$\sin(x)$	$\cos(x)$	$-\cos(x)$
$\cos(x)$	$-\sin(x)$	$\sin(x)$

PHYSICS & MEASUREMENT

A. **Understand Your Data**

1. **Vector vs. scalar**
   a. **Vector**: Has magnitude **and** direction
   b. **Scalar**: Magnitude only, no direction

2. **Number and unit**
   a. Physical data, constants and equations have numerical values and units
   b. A correct answer must include the correct numerical value **PLUS** the correct unit

3. **Significant figures (sigfig)**
   a. The # of sigfigs reflects the accuracy of experimental data; calculations must accommodate this uncertainty
   b. **For multiplication**: The # of sigfigs in the final answer is limited by the entry with the fewest sigfigs
   c. **For addition**: The # of decimal places in the final answer is given by the entry with the fewest decimal places
   d. **Rules for "rounding sigfigs"**
      - If the last digit is >5, round up
      - If the last digit is <5, round down
      - If digit = 5, round up if preceding digit is odd

   ✎ **Samples**:
   1.245 + 0.4 = 1.6 (1 decimal place)
   1.345 × 2.4 = 3.2 (2 sigfigs)

Units of basic variables

time: second s	position: meter m
mass: kilogram kg	volume: m^3
density: kg/m^3	Temperature: K
velocity: m/s	acceleration: m/s^2
energy: Joule J = $kg\ m^2/s^2$	force: Newton N = $kg\ m/s^2$

⚠ **Pitfall**: If the units are wrong, the answer is wrong!

🔎 **Hint**: **Before** doing the calculation:
* Check **all** constants and variable units
* Take special care if you derive the equation

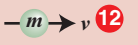
4. **Dimensional analysis**

Verify that constants and variables in an equation result in the correct overall unit

✎ **Samples**: The energy unit is Joules for kinetic, gravitational and Coulombic energy

- **Kinetic**: $K = \frac{1}{2}mv^2$

m in kg, v in m/s Units of K = kg $m^2/s^2 = J$ fig 12

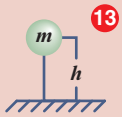


12

- **Gravitational** potential energy: $U_g = m\ g\ h$ Constant: $g = 9.8\ m/s^2$

m in kg, h in m

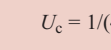
Units of U_g = kg $m^2/s^2 = J$ fig 13



13

- **Electrostatic** potential energy:

$U_c = 1/(4\pi\varepsilon_0)\frac{q_1 q_2}{r}$



14

constant: $1/4\pi\varepsilon_0$; units are J m/C^2

r in m, q_1 and q_2 in Coulomb

Units of U_c = (J m/C^2)C^2/m = J fig 14

5. **Using Conversion Factors**

a. **Purpose**: Modify experimental data to match the units of constants and equations

b. **SI units**: MKS (m-kg-s) and CGS (cm-g-s)

c. Common English units: Foot, pounds, BTU, calories

d. Conversion factors are obtained from an **equality** of two units

✎ **Sample**: 100 cm = 1 m
- This equality gives two conversion factors:

$\frac{1m}{100cm}$ & $\frac{100cm}{1m}$

- Use the 1st factor to convert "cm" to "m"

✎ **Sample**: 54 cm × $\frac{1m}{100cm}$ = 0.54 m
- Use the 2nd to convert "m" to "cm"

✎ **Sample**: 2.3 m × $\frac{100cm}{1m}$ = 230 cm

B. **Solve the Problem Strategically**

a. **Two key issues**:
1. Understand the physics principles
2. Have a correct mathematical strategy

b. **Useful steps in problem solving**:
1. Prepare a rough sketch of the problem
2. Identify relevant physical variables, physical concepts and constants

⚠ **Pitfall** - do not simply search for the "right" equation in your notes or text

3. Describe the physics using a mathematical diagram, with appropriate symbols and a coordinate system
4. Obtain the relevant physical constants
Do you have all the essential data?

🔎 **Hint**: You may have extra information

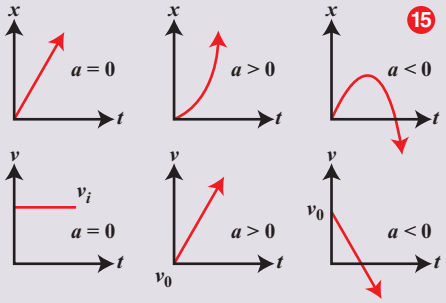
5. The **hard part**: Derive or obtain a mathematical expression for the problem; use dimensional analysis to check the equation, constants and data
6. The **easy part**: Plug numbers into the equation and use the calculator to obtain the numerical answer
7. Check the final answer, using the original statement of the problem, your sketch and common sense; are the units & sign correct?

MECHANICS

A. **Motion along a Straight Line**

1. **Goal**: Determine position, velocity, acceleration
2. **Key terms**: Acceleration: $a = dv/dt$; velocity: $v = dx/dt$
3. **Key Equations**: $x = v_i t + \frac{1}{2}at^2$ $v_f = v_i + at$

x*(*t*), *v*(*t*) for variable *a fig 15



15

B. **Motion in Two and Three Dimensions**

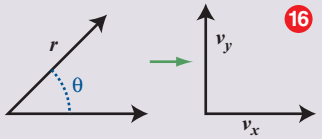
1. **Goal**: Similar to "*A*," with 2 or 3 dimensions
2. **Key concept**: Select Cartesian, polar or spherical coordinates, depending on the type of motion

Sample: A projectile is launched at angle θ with v_{ri}; how do we set up the problem?

Step 1. Define x as horizontal and y vertical

Step 2. Determine initial v_{xi} and v_{yi} fig 16

$v_{xi} = v_{ri}\cos\theta$ $v_{yi} = v_{ri}\sin\theta$



16

Step 3. Identify a_x - Gravitational force => $a_y = -g$

Step 4. Identify a_y - No horizontal force => $a_x = 0$

Step 5. Develop x- and y-equations of motion

$$x = v_{ix}t + \frac{1}{2}a_x t^2 = v_i t$$

$$y = v_{iy}t + \frac{1}{2}a_y t^2 = v_{iy}t - \frac{1}{2}gt^2$$

C. Newton's Laws of Motion

1. **Goal**: Examine force and acceleration
2. **Key concepts**: **Newton's Laws**:

Law #1. A body remains at rest or in motion unless influenced by a force

Law #2. Forces acting on a body equal the mass multiplied by the acceleration; force and acceleration determine motion

Law #3. Every action is countered by an opposing action

3. **Key equations**:

a. **Law #2**: $\boldsymbol{F} = m\,\boldsymbol{a}$ or $\Sigma\boldsymbol{F} = m\,\boldsymbol{a}$

Hint: Forces are vectors!

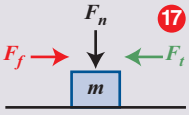
b. **Types of forces**: **Body** - gravity: $F_g = m\,\boldsymbol{g}$

- **Surface** - friction: $= F_f = \mu F_n$

Sample: F_t exerted on object on a horizontal plane

$F_f = \mu F_n = \mu F_g = \mu\, m\, g$

Net force = $F_t - F_f$ fig 17



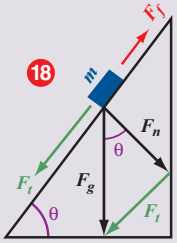
Sample: Object on plane inclined at angle θ;

examine F_g & F_f

$F_n = F_g \cos\theta = m\,g\cos\theta$

$F_f = \mu F_n = \mu\, m\, g \cos\theta$

$F_t = m\,g\,\sin\theta$ fig 18



e. **Law #3**:

$F_{12} = -F_{21}$ or $m_1 a_1 = -m_2 a_2$

Sample: Examine recoil of bullet fired from a rifle

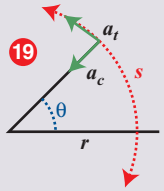
Rifle recoil = a(bullet) × m(bullet)

D. Circular Motion fig 19

1. **Goal**: Examine body moving in a circular path; use 2-d polar coordinates: (r, θ)

Key variables:

r	m	distance from rotation center
θ	rad	angle with reference (x) axis
ω	rad/s	angular velocity
α	rad/s^2	angular acceleration
s	m	motion arc; $s = r\theta$ (θ in rad)



Hint: For a full rotation, $s = 2\pi r$ = circumference of a circle of radius r

2. **Tangential** acceleration & velocity: $v_t = r\omega$; $a_t = r\alpha$; along path of motion arc
3. **Centripetal** acceleration: $a_c = \frac{v^2}{r}$; directed towards the center fig 19

Sample: Determine v_t at the Earth's equator

Equation: $v_t = r\omega$ **Data**: $r = 6.378 \times 10^6$ m

$\omega = 2\pi$ rad/day; 1 day = 24 × 60 × 60 sec = 86,400 s

Convert ω to SI: ω = 2π rad/day × 1 day/86,400 s = 7.3×10^{-5} rad/s

Calculate v_t:

$v_t = r\omega = 6.378 \times 10^6$ m × 7.3×10^{-5} rad/s v_t = 465 m/s

E. Energy and Work

1. **Goal**: Examine the energy and work associated with forces acting on an object
2. **Key equations**:

a. Kinetic energy: $\frac{1}{2}mv^2$; energy of motion

b. Work: Force acting over a distance

- For $F(x)$: Work = $\int F(x)\,dx$
- For a constant force: $W = F\,d\cos\theta = \boldsymbol{F} \times \boldsymbol{r}$
- θ is the angle between the F and r
- W maximum for θ = 0 (note: sin(θ = 0) = 1)

c. **Power = Work/time**: W = PowerΔt or $\int P(t)\,dt$

d. $\boldsymbol{W_{net} = K_{final} - K_{initial}}$; K is converted to work

Sample: Determine the work expended in lifting a 50 kg box 10 m; given: $a = g = 9.8$ m/s^2

Equations: $F = m\,g$ => W = $m\,g\,d$

Calculation: $W = 50$ kg × 9.8 m/s^2 × 10 m = 4,900 J

F. Potential Energy & Energy Conservation

1. **Goal**: Use **energy conservation** to study the interplay of potential and kinetic energy
2. **Key Equations**

a. **Potential energy**: Energy of position: $U(r)$; gravitation ($U = mgh$), electrostatic ($U \propto qq/r$)

b. $\boldsymbol{E = K + U}$ Conservative system: No external force

Sample: Examine K & U for a launched rocket

Initial: $h = 0$, therefore, $U = m\,g\,h = 0$ ❶

$E = K_i = \frac{1}{2}m\,v_i^2$

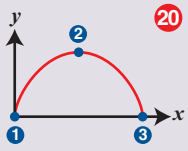
Next, resolve into x and y components: K_{xi} & K_{yi}

Note: K_x is constant during the flight

At max height: $K_y = 0$; $U = m\,g\,h = K_{xi}$ ❷

Final state: Rocket hits the ground: $U = 0$, $K = K_i$ ❸

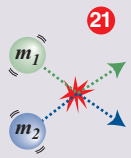
fig 20



G. Collisions and Linear Momentum fig 21

1. **Goal**: Examine momentum of colliding bodies

Hint: For 2-D or 3-D, use Cartesian components



2. **Key Variables and Equations**

a. **Types of collisions**:

- **Elastic**: Conserve energy
- **Inelastic**: Energy lost as heat or deformation

b. **Relative motion and frames of reference**: A body moves with velocity v in frame S; in frame S', the velocity is v'; if V_s, is the velocity of frame S' relative to S, then $\boldsymbol{v = V_s + v'}$

c. **Linear Momentum**: $\boldsymbol{p = m\,v}$

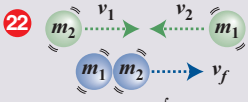
d. Conserve K & $\boldsymbol{p}$ for **conservative** system (no external forces):

$$\Sigma\tfrac{1}{2}m{v_i}^2 = \Sigma\tfrac{1}{2}m{v_f}^2 \qquad \Sigma m v_i = \Sigma m v_f$$

Sample 1-d problem: Two bodies collide, stick together and move away from the collision site fig 22

Conservation of momentum:

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)v_f$$



f. **Impulse:** $I = F\Delta t$ or $\int F(t)dt$

g. **Momentum change**: $p_{fin} = p_{init} + I$

H. Rotation of a Rigid Object

1. **Goal**: Examine the rotation of a rigid body of a defined shape and mass
2. **Key variables and equations**:

a. **Center of mass**: x_{cm}, y_{cm}, z_{cm}

$$x_{cm} = \frac{\Sigma m_i x_i}{\Sigma m_i} \qquad y_{cm} = \frac{\Sigma m_i y_i}{\Sigma m_i} \qquad z_{cm} = \frac{\Sigma m_i z_i}{\Sigma m_i}$$

Sample: Calculate the **center of mass** for a 1 kg & a 2 kg ball connected by a 1.00 m bar

ball 1: $x_1 = 0.00$, $m_1 = 1$ kg; $m_1 x_1 = 0.00$ kg m

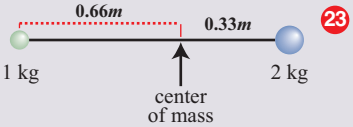
ball 2: $x_2 = 1.00$ m $_2 = 2$ kg; $m_2 x_2 = 2.00$ kg m

$\Sigma m_i = 1$ kg + 2 kg = 3 kg

$\Sigma m_i x_i = m_1 x_1 + m_2 x_2 = 0.00 + 2.00 = 2.00$ kg m

$$x_{cm} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{2.00 kg\, m}{3.00 kg} = 0.66\ m$$

Hint: The center of mass is nearer the heavier ball fig 23



b. **Moment of inertia**:

$I = \Sigma m_i r_i^2$, with r_i about the center of mass **along a specific axis**

Hint: I functions as the **effective mass** for rotational energy and momentum

Sample: $\boldsymbol{I}$ for bodies of mass m: fig 24

Twirling thin rod of length, L

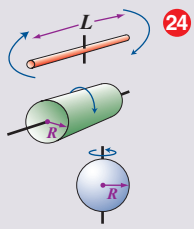
$$\mathbf{I} = \frac{1}{12}\,\boldsymbol{m\,L^2}$$

Rotating cylinder of radius, R

$$\mathbf{I} = \frac{1}{2}\,\boldsymbol{m\,R^2}$$

Rotating sphere of radius, R

$$\mathbf{I} = \frac{2}{5}\,\boldsymbol{m\,R^2}$$



Sample: Determine the I for a spherical Earth, assume uniform M;

Data: $M = 6\times10^{24}$ kg, $r = 6.4\times10^6$ m

$I = \frac{2}{5}M\,r^2 = \frac{2}{5} \times 6\times10^{24}$ kg × $(6.4\times10^6$ m$)^2$

$= 9.8 \times 10^{37}$ kg m^2

e. **Rotational Energy** = $\frac{1}{2}I\,\omega^2$

f. **Torque**: $\boldsymbol{\tau} = I\omega = \boldsymbol{r} \times \boldsymbol{F}$ (ang. acceleration force)

I. Angular Momentum

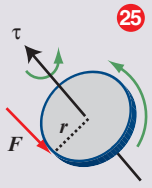
1. **Goal**: Quantify the force, energy and momentum of rotating objects
2. **Key variables and equations**

a. **Angular momentum**:

$\boldsymbol{L} = I\omega = \boldsymbol{r} \times \boldsymbol{p} = \int \boldsymbol{r} \times \boldsymbol{v} dm$

b. **Torque**: $\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F} = d\boldsymbol{L}/dt$;

note: vector cross product fig 25



J. Static Equilibrium and Elasticity

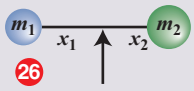
1. **Case 1**: Examine several forces acting on a body

- **Guiding principles**: Equilibrium is defined as: Σforce = 0 & Στorque = 0

The **point of balance** is the center of mass

Hint: Evaluate each component; any net force moves the object, any net torque rotates the object

Sample: Beam balance fig 26



For equilibrium:

$\boldsymbol{m_1 x_1 = m_2 x_2}$

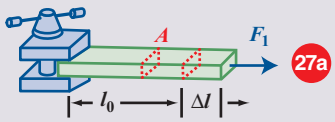
2. **Case 2.** Examine deformation of a solid body

Key Equation: Stress = elastic modulus × strain; **modulus**: stress/strain = force/change (Hooke's Law)

- **Linear (Tensile) Stress**: Young's Modulus Y

$$Y = \frac{F_\perp / A}{\Delta l / l_0}$$

fig 27a

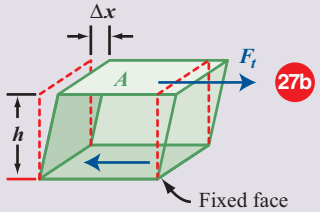


Note: Force $\boldsymbol{F}_\perp$ is longitudinal

- **Shape Stress**: Shear Modulus S

$$S = \frac{F_t / A}{\Delta x / h}$$

fig 27b

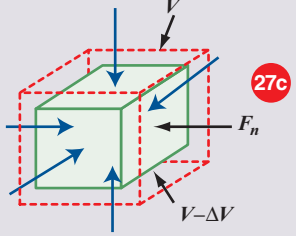


Note: Force F_t is tangential to face A

- **Volume Stress**: Bulk Modulus B

$$B = \frac{F_n / A}{\Delta V / V}$$

fig 27c

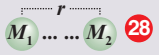


Note: Force F_n is normal to face A

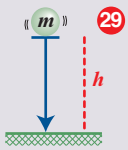
MECHANICS (*continued*)

K. Universal Gravitation

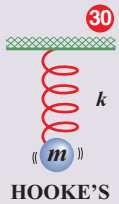
1. **Goal**: Examine gravitational energy and force fig 28
2. **Case 1**: Bodies of mass M_1 & M_2 separated by r



3. **Key equations**:
   a. Gravitational **Energy**: $U_g = \frac{GM_1M_2}{r}$
   b. Gravitational **force**: $F_g = \frac{GM_1M_2}{r^2}$
   c. **Acceleration due to gravity**: $g = G\,M(\text{earth})/r^2$
   For objects on the Earth's surface, $g = 9.8\ \text{m/s}^2$

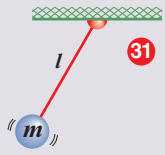


   ✎ **Sample**: Verify "g" **at the Earth's surface**
   Equation: $g = G\,M(\text{earth})/r^2$
   Given: M = 6×10^{24} k, $r = 6.4\times10^6$ m
   Calculation: $= \frac{6.67\times10^{-11}\,m^3\,kg^{-1}\times6\times10^{24}\,kg}{(6.4\times10^6\,m)^2} = 9.8\ \text{ms}^2$

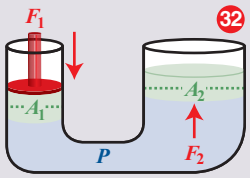


HOOKE'S LAW

4. **Case 2**: A body interacts with the Earth fig 29
5. **Key Equation**:
   a. **Gravitational potential energy**: $U_g = m\,g\,h$; object on the Earth's surface, $h = 0$; $U_g = 0$
   b. **Weight** = gravitational force; $F_g = m\,g$

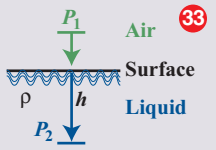


   ✎ **Sample**: Calculate **escape velocity**, v_{esc}, for an orbiting rocket of mass m at altitude h
   🔎 **Hint:** $K = U_g$ at point of escape; $r = h + r$(earth)
   $\frac{1}{2}m\,v_{esc}^2 = \frac{GmM}{r}$; therefore, $v_{esc} = \sqrt{\frac{2GM}{r}}$
   Note: v_{esc} varies with altitude, but not rocket mass

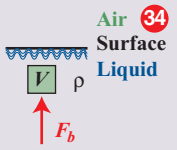


L. Oscillatory Motion

1. **Goal**: Study motion & energy of oscillating body
2. **Simple harmonic motion** (1-d)
   a. Force: $F = -k\Delta x$ (Hooke's Law)
   b. Potential Energy: $U_k = \frac{1}{2}k\Delta x^2$
   c. Frequency = $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ fig 30



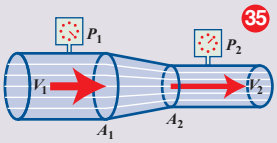
3. **Simple Pendulum**
   a. Period: $T = 2\pi\sqrt{\frac{l}{g}}$
   b. Potential energy: $U_g = m\,g\,h$
   c. Frequency = $\frac{1}{2\pi}\sqrt{\frac{g}{l}}$ fig 31



4. **For both cases**:
   a. **Kinetic energy**: $K = \frac{1}{2}m\,v^2$
   b. **Conservation of Energy**: $E = U + K$

M. Forces in Solids and Liquids

1. **Goal 1**: Examine properties of solids & liquids
   a. **Density of a solid or liquid**: $\rho = \frac{\text{mass}}{\text{volume}}$
   • Common unit: g/cm^3; g/L; kg/m^3
   ▪ **Sample**: A piece of metal, 1.5 cm × 2.5 cm × 4.0 cm, has a mass of 105.0 g; determine ρ
   Equation: $\rho = \frac{m}{V}$
   Data: $m = 105.0$ g, $V = 1.5\times2.5\times4.0\ \text{cm}^3 = 15\ \text{cm}^3$
   Calculate: $\rho = 105.0/15.0\ \text{g/cm}^3 = 7.0\ \text{g/cm}^3$



Flow Through a Hose

   b. Pressure exerted by a fluid: $P = \frac{\text{force}}{\text{area}}$
   c. **Pascals's Law**: For an enclosed fluid, pressure is equal at all points in the vessel
   ▪ **Sample**: **Hydraulic press**: $F = P/A$ for enclosed liquid; A is the surface area of the piston inserted into the fluid
   Equation: $A_1F_1 = A_2\,F_2$; cylinder area determines force fig 32
   d. A column of water generates pressure, P increases with depth;
   Equation: $P_2 = P_1 + \rho gh$ fig 33
   e. **Archimedes' Principle**: Buoyant force, F_b, on a object of volume V submerged in liquid of density ρ: $F_b = \rho Vg$ fig 34

2. **Goal 2**: Examine fluid motion & fluid dynamics
   a. Properties of an **Ideal fluid**: Non-viscous, incompressible, steady flow, no turbulence At any point in the flow, the product of area and velocity is constant: $A_1v_1 = A_2v_2$
   b. Variable density: $\rho_1A_1v_1 = \rho_2A_2v_2$; illustrations: gas flow through a smokestack, water flow through a hose fig 35
   c. **Bernoulli's Equation**: For any point y in the fluid flow, $P + \frac{1}{2}\rho v^2 + \rho\,g\,y = \text{constant}$
   • **Special case**: Fluid at rest $P_1 - P_2 = \rho\,g\,h$

WAVE MOTION

A. Descriptive Variables

1. **Types**: Transverse, longitudinal, traveling, standing, harmonic
   a. General form for transverse traveling wave: $y = f(x - vt)$ (to the right) or $y = f(x + vt)$ (to the left)
   b. General form of **harmonic wave**: $y = A\sin(kx - \omega t)$ **or** $y = A\cos(kx - \omega t)$
   c. **Standing wave**: Integral multiples of $\frac{\lambda}{2}$ fit the length of the oscillating material
   d. **General wave equation**: $\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$
   e. **Superposition Principle**: Overlapping waves interact => constructive and destructive interference

Harmonic Wave Properties

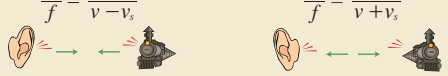
Wavelength	λ (m)	Distance between peaks
Period	T (sec)	Time to travel one λ
Frequency	f (Hz)	$f = \frac{1}{T}$
Angular Frequency	ω (rad/s)	$\omega = \frac{2\pi}{T} = 2\pi f$
Wave Amplitude	A	Height of wave
Speed	v (m/s)	$v = \lambda f$
Wave number	k (m^{-1})	$k = \frac{2\pi}{\lambda}$

2. ✎ **Sample**: Determine the velocity and period of a wave with λ = 5.2 m and f = 50.0 Hz
   Equations: $v = \lambda f$ $\quad T = \frac{1}{f}$
   Data: λ = 5.20 m; f = 50.0 Hz
   Calculations: $v = \lambda f = 5.20\ \text{m} \times 50.0 = 260\ \text{m/s}$
   $T = \frac{1}{f} = \frac{1}{50}\ \text{Hz} = 0.02\ \text{s}$

B. Sound Waves

1. **Wave nature of sound**: Compression wave displaces the medium carrying the wave
2. **General speed of sound**: $v = \sqrt{\frac{B}{\rho}}$;
   note: B = **Bulk Modulus** (measure of volume compressibility)
   For a gas: $v = \sqrt{\frac{\gamma RT}{M}}$; note: $\gamma = \frac{C_p}{C_v}$ (ratio of gas heat capacities)
   ✎ **Sample**: Calculate speed of sound in Helium at 273 K
   Helium: Ideal gas, γ = 1.66; M = 0.004 kg/mole
   $v = \sqrt{\frac{\gamma RT}{M}}$
   $= \sqrt{\frac{1.66\times8.314\ \text{kg m}^2/\text{s}^2\times273\text{K}}{0.004\text{kg}}}$
   $= \sqrt{941{,}900\ \text{m}^2/\text{s}^2} = 971\ \text{m/s}$ **note:** $\sqrt{}$ applies to the units
3. **Loudness as intensity and relative intensity**
   a. **Absolute Intensity** (I = Power/Area) is an inconvenient measure of loudness
   b. Relative loudness: **Decibel scale (dB)**: $\beta = 10\log\frac{I}{I_0}$; I_0 is the threshold of hearing; $\beta(I_0) = 0$
   c. **Samples**: Jet plane: 150 dB; Conversation: 50 dB; a change in 10 dB represents a 10–fold increase in I
4. **Doppler effect**: The sound frequency shifts $\frac{f'}{f}$ due to relative motion of source and listener;
   v_0 - listener speed; v_s - source speed; v - speed of sound

$\frac{f'}{f} = \frac{v+v_0}{v-v_s}$ $\qquad$ $\frac{f'}{f} = \frac{v-v_0}{v+v_s}$



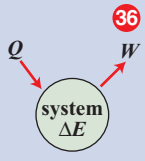
Key: Identify **relative** speed of source and listener

THERMODYNAMICS

A. Goal: Study of work, heat and energy of a system fig 36

Key Variables

Heat: Q	$+Q$ added to the system
Work: W	$+W$ done by the system
Energy: E	System internal E
Enthalpy: H	$H = E + PV$
Entropy: S	Thermal disorder
Temperature: T	Measure of thermal E
Pressure: P	Force exerted by a gas
Volume: V	Space occupied



THERMODYNAMICS (*continued*)

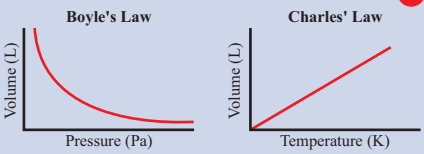
Types of Processes

Isothermal	$\Delta T = 0$	$\Delta E = 0$, $Q = W$ PV = constant
Adiabatic	$Q = 0$	$\Delta E = -W$ PV^{γ} = constant
Isobaric:	$\Delta P = 0$	$W = P\Delta V$, $\Delta H = Q$
Isochoric	$\Delta V = 0$	$\Delta E = Q$; $W = 0$

B. Temperature & Thermal Energy

1. **Goal**: Temperature is **in Kelvin, absolute temperature**: T**(K) =** T**(°C) + 273.15**
   Note: T(K) is always positive; lab temperature must be converted from °C to Kelvin (K)
   ✎ **Sample**: Convert 35° C to Kelvin:
   T(K) = T(°C) + 273.15 = 35 + 273.15 = 308.15 K
2. **Thermal Expansion of Solid, Liquid or Gas**
   a. **Goal**: Determine the change in the length (L) or volume (V) as a function of temperature
   b. **Solid**: $\frac{\Delta L}{L} = \alpha \Delta T$
   c. **Liquid**: $\frac{\Delta V}{V} = \beta \Delta T$
   d. **Gas**: $\Delta V = \frac{(T_2 - T_1)nR}{P}$
3. **Heat capacity**: $C = \frac{Q}{\Delta T}$ **or** $Q = C\Delta T$
   a. **Special cases**: C_p–constant P; C_v–constant V
   • **Ideal Gas**:
   $C_p = \frac{5}{2}R$; $C_v = \frac{3}{2}R$; $\gamma = \frac{C_p}{C_v} = \frac{5}{3} = 1.667$
   b. **Carnot's Law**: For ideal gas: $C_p - C_v = R$
   • $\Delta E = C_v \Delta T$; $\Delta H = C_p \Delta T$
   • Exact for monatomic gas, modify for molecular gases

37



C. Ideal Gas Law; $PV = nRT$ fig 37

1. **Goal**: Simple equation of state for a gas
2. **Key Variables**: P(Pa), V(m^3), T(K), n moles of gas (mol); gas constant R = 8.314 J mol^{-1} K^{-1}
   ⚠ **Pitfall**: Common errors in T, P or V units
3. **Key Applications**:
   a. $P \propto \frac{1}{V}$, T fixed: Boyle's Law
   b. $P \propto T$, V fixed
   c. $V \propto T$, P fixed: Charles' Law
   d. Derive thermodynamic relationships

D. Enthalpy and 1st Law of Thermodynamics

1. **Goal**: Determine Q, ΔE and W; W and Q depend on path; ΔE is a state variable, independent of path
2. **Guiding Principle**:
   a. **1st Law of Thermodynamics**: $\Delta E = Q - W$
   • Key idea: Conservation of Energy
   b. Examine the T, P, W & Q for the problem
3. **Enthalpy**: $H = E + PV$; $\Delta H = \Delta E + P\Delta V$
   a. $\Delta H = Q$ for $\Delta P = 0$ (constant pressure)
   b. Variable temperature: $\Delta H = \int C_p \, dT$
   c. For constant C_p: $\Delta H = C_p \Delta T$
4. **Work**: $W = \int P dV$
   a. W depends on the path or process
   b. Ideal Gas, Reversible, Isothermal:
   $W = nRT \ln \frac{V_2}{V_1}$
   c. Ideal Gas, Isobaric: $W = P\Delta V$

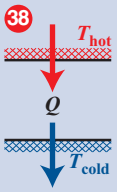
E. The Kinetic Theory of Gases

1. **Goal**: Examine kinetic energy of gas molecules
2. **Key Equations**: $E = \frac{1}{2}Mv^2$ and $E = \frac{3}{2}RT$
   a. Speed, $v_{rms} = \sqrt{\frac{3RT}{M}}$
   ✎ **Sample**: Calculate the speed of Helium at 273 K
   Helium: M = 0.004 kg/mole
   $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.314 \text{ kg m}^2/\text{s}^2 \times 273 \text{ K}}{0.004 \text{ kg}}}$
   v_{rms} = $\sqrt{1,702,292}$ **m/s = 1305 m/s**
   b. **Kinetic energy for Ideal Gas**: $K = \frac{3}{2}RT$
   c. **For real gas**: Add terms for vibrations and rotations

F. Entropy & 2nd Law of Thermodynamics

1. **Goal**: Examine the driving force for a process
2. **Key Variables**:
   a. **Entropy**: S, thermal disorder; $dS = \frac{dQ}{T}$
   b. S(univ) = S(system) + S(thermal bath)
3. **Guiding Principle**: 2nd Law of Thermodynamics:
   For any process, $\Delta S_{univ} > 0$; one exception:
   $\Delta S_{univ} = 0$ for a reversible process
4. **Examples**:
   a. Natural heat flow: Q flows from T_{hot} to T_{cold} fig 38
   $\Delta S_{univ} = \Delta S_{hot} + \Delta S_{cold} = -\frac{Q}{T_{hot}} + \frac{Q}{T_{cold}} = Q\frac{T_{hot} - T_{cold}}{T_{hot}\,T_{cold}}$
   🔎 **hint**: $\Delta S_{univ} > 0$ for a natural process

38

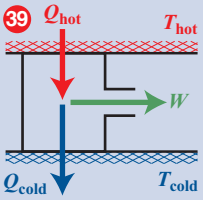


   b. **Phase change**: $\Delta S = \frac{Q(phase\ change)}{T(phase\ change)}$
   c. Ideal Gas $S(T)$: $\Delta S = nC_p \ln \frac{T_2}{T_1}$
   d. Ideal Gas: $S(V)$: $\Delta S = nR \ln \frac{V_2}{V_1}$

G. Heat Engines

1. **Goal:** Examine Q and W of an engine
2. **Thermal Engine**: The engine transfers Q from a hot to a cold reservoir, and produces W fig 39
3. Efficiency of engine: $\xi = \frac{W}{Q_{hot}} = 1 - \frac{Q_{cold}}{Q_{hot}}$

39



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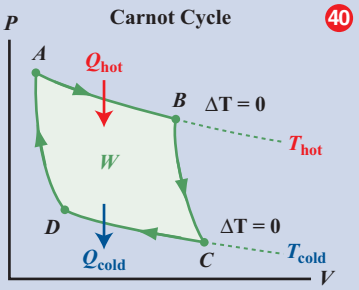


4. **Idealized** heat engine: **Carnot Cycle** fig 40
   a. Four steps in the cycle: two isothermal, two adiabatic; for overall cycle: $\Delta E = 0$ and $\Delta S = 0$
   b. Efficiency = $1 - \frac{T_{cold}}{T_{hot}}$

ELECTRICITY & MAGNETISM

A. Electric Fields and Electric Charge

1. **Goal**: Examine the nature of the field generated by an electric charge, and forces between charges
2. **Key Variables and Equations**
   a. **Coulomb** C: "ampere sec" of charge
   b. e - charge on an electron; 1.6022×10^{-19} C
   c. **Coulomb' Law** - electrostatic force: $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\boldsymbol{e}$
   • Vector direction defined by $\boldsymbol{e}$
   d. **Electric Field**: $E = \frac{F}{q}$
   🔎 **Hint**: **Calculation shortcut**:
   $F = 9 \times 10^9 N \frac{q_1(C) q_2(C)}{r(m)^2}$
   Note: q in Coulombs and r in meters
3. **Superposition Principle**: Forces and fields are composites of contributions from each charge
   $F = \Sigma F_i$, $E = \Sigma E_i$; 🔎 **Hint**: Forces and electric fields are vectors

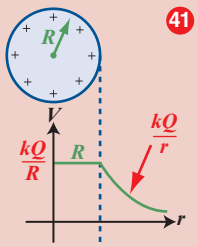
B. Gauss's Law

1. **Goal**: Define **electric flux**, Φ_e
2. **Key Variables and Equations**:
   a. **Gauss's Law**: $\Phi_e = \oint E \times dA = \frac{Q}{\varepsilon_0}$
   b. The electric flux, Φ_e, depends on the **total charge** in the closed region of interest

C. Electric Potential & Coulombic Energy

1. **Goal**: Determine **Coulombic** potential energy
2. **Key Variables and Equations**:
   a. **Potential energy**: $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$
   b. **Potential**: $V(q_1) = \frac{U}{q_2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$
   Note: The potential is scalar, depending on $|r|$
   c. For an array of charges, q_i, $V_{total} = \Sigma V_i$
   d. **Shortcut to $U(r)$**: $U = 9 \times 10^9 \text{J} \frac{q_1(C) q_2(C)}{r(m)}$
3. **Continuous charge distributions**: $V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$ fig 41
   ✎ **Sample**: **Conducting sphere**, Radius R, Charge Q
   $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ for $r \le R$
   $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ for $r > R$

41



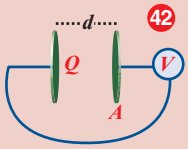
Conducting Sphere

4. **Dielectric effect**: V & F depend on the **dielectric constant, κ**; replace ε_0 with $\kappa\varepsilon_0$ for the material;
   $V(\kappa) = \frac{1}{\kappa}V(vacuum)$
   $F(\kappa) = \frac{1}{\kappa}F(vacuum)$

D. Capacitance and Dielectrics

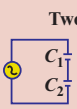
1. **Goal**: Study capacitors, plates with charge Q separated by a vacuum or dielectric material fig 42
2. **Key Equations**:
   a. **Capacitance**, $C = \frac{Q}{V}$, V is the measured voltage
   b. Parallel plate capacitor, vacuum, with area A, spacing d: $C = \varepsilon_0 \frac{A}{d}$; $\boldsymbol{E} = \frac{Q}{\varepsilon_0}A$
   c. Parallel plate capacitor, dielectric κ, with area A, spacing d: $C = \kappa\varepsilon_0 \frac{A}{d}$
3. **Capacitors in series**: $\frac{1}{C_{tot}} = \sum \frac{1}{C_i}$
4. **Capacitors in parallel**: $C_{tot} = \Sigma C_i$ fig 43

42



43

Two Capacitors in Series



$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2}$ *or* $C_{tot} = \frac{C_1 C_2}{C_1 + C_2}$

Two Capacitors in Parallel

$C_{tot} = C_1 + C_2$

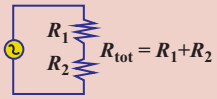
ELECTRICITY & MAGNETISM (continued)

E. Current and Resistance

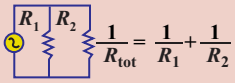
1. **Goal**: Examine the current, I, quantity of charge, Q, resistance, R; determine the voltage and power dissipated
2. **Key Equations**:
   a. **Total charge**, $Q = It$
   b. $V = IR$, or $R = \frac{V}{I}$
   c. **Resistors in Series**: $R_{tot} = \Sigma R_i$ fig 44
   d. **Resistors in Parallel**: $\frac{1}{R_{tot}} = \Sigma \frac{1}{R_i}$ fig 44
   e. **Power** $= IV = I^2R$

44

Two Resistors in Series



Two Resistors in Parallel

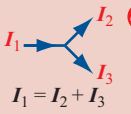


or $R_{tot} = \frac{R_1 R_2}{R_1 + R_2}$

F. Direct Current Circuit

1. **Goal**: Examine a circuit containing battery, resistors and capacitors; determine voltage and current properties
2. **Key Equations and Concepts**:
   a. EMF: Circuit voltage; $\mathcal{E} = V_b + IR$; battery voltage $V_b = Ir$, r internal battery resistance
   b. **Junction**: Connection of 3 or more conductors
   c. **Loop**: A closed conductor path
   d. Resistors in series or parallel => replace with R_{tot}
   e. Capacitors in series or parallel => replace with C_{tot}
3. **Kirchoff's Circuit Rules**
   a. For any Loop: $\Sigma V = \Sigma IR$;
   Hint: Conserve energy
   b. For any Junction: $\Sigma I = 0$;
   Hint: Conserve charge; define "+" flow fig 45

45



G. Magnetic Field: *B*

1. **Key concepts:**
   a. Moving charge => Magnetic Field B
   b. **Magnetic Flux**: $\Phi_m = \oint B\,dA$
   c. Force on charge, q and v, moving in B: $F = qv \times B = qvB\sin\theta$; v parallel to B => $F = 0$; v perpendicular to B => $F = qvB$
   d. Magnetic Moment of a Loop: $M = IA$
   e. Torque on a loop: $\tau = M \times B$
   f. Magnetic **Potential Energy**: $U = -M \cdot B$
   g. **Lorentz Force**: Charge interacts with E and B; $F = qE + qv \times B$

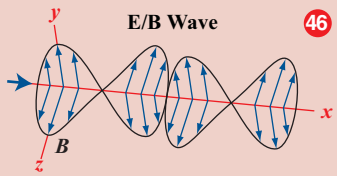
H. Faraday's Law and Electromagnetic Induction - Key Equations:

1. **Faraday's Law**: Induced EMF: $\mathcal{E} = \oint E\,ds = -d\Phi_m/dt$
2. **Biot-Savart Law**: Conductor induces B; current I, length dL: $dB = \frac{\mu_0}{4\pi} I dL \times \frac{r}{r^3}$
3. **Sample**: Long conducting wire: $B(r) = \frac{\mu_0}{4\pi}\frac{I}{r}$

I. Electromagnetic Waves- Key Equations and Concepts:

1. Transverse B and E fields; $\frac{E}{B} = c$

E/B Wave 46



2. $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$
3. Electromagnetic Wave: $c = f\lambda$ fig 46

J. Maxwell's Equations:

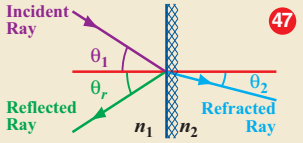
1. **Gauss's Law for Electrostatics**: $\oint E \cdot dA = \frac{Q}{\varepsilon_0}$; **key**: Charge gives rise to E
2. **Gauss's Law for Magnetism**: $\oint B \cdot dA = 0$; **key**: Absence of magnetic charge
3. **Ampere-Maxwell Law**: $\oint B \cdot ds = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$; **key**: Current + change in electric flux => B
4. **Faraday's Law**: $\oint E \cdot dS = -\frac{d\Phi_m}{dt}$;

BEHAVIOR OF LIGHT

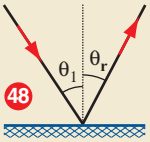
A. Basic Properties of Light

1. **Goal**: Examine light and its interaction with matter
2. **Key variables**:
   a. c: speed of light in a vacuum
   b. **Index of refraction**: n; $\frac{c}{n}$ = speed of light in medium
   c. Light as **electromagnetic wave**: $\lambda f = c$
   Light characterized by "color" or wavelength
   d. **Light as particle**: $e = hf$; energy of photon
3. **Reflection and Refraction of Light** fig 47
   a. **Law of Reflection**: $\theta_1 = \theta_r$ fig 48
   b. **Refraction**: Bending of light ray as it passes from n_1 to n_2
   • **Snell's Law**: $n_1\sin\theta_1 = n_2\sin\theta_2$, n_1, n_2 are the indices of refraction of two materials fig 49
   c. **Internal Reflectance**: $\sin\theta_c = \frac{n_2}{n_1}$
   Light passing from material of higher ***n*** to a lower ***n*** may be trapped in the material
4. **Polarized light**: ***E*** field is not spherically symmetric
   a. **Examples**: Plane/linear polarized, circularly polarized
   b. Polarization by reflection from a dielectric surface at angle θ_c;
   Brewster's Law: $\tan\theta_c = \frac{n_2}{n_1}$

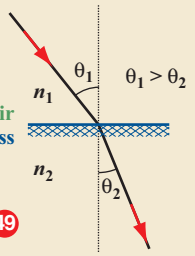
Reflection and Refraction 47



Normal 48



Normal 49



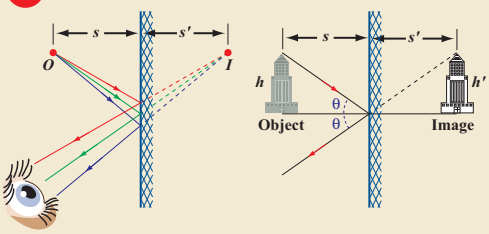
B. Lenses and Optical Instruments

1. **Goal**: Lenses and mirrors generate images of objects
2. **Key concepts and variables**
   a. Radius of curvature: $R = 2f$
   b. **Optic axis**: Line from base of object through center of lens or mirror
   c. **Magnification**: $M = \frac{s'}{s}$
   d. **Laws of Geometric Optics**: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$; $\frac{s}{s'} = -\frac{h}{h'}$
   e. Combination of 2 thin lenses: $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ or $f = \frac{f_1 f_2}{f_1 + f_2}$
3. **Sample Guidelines for ray tracing:**
   a. Rays that parallel optic axis pass through "f"
   b. Rays pass through center of the lens unchanged
   c. Image forms at convergence of ray tracings
   Sample ray tracings: fig 50, a,b,c

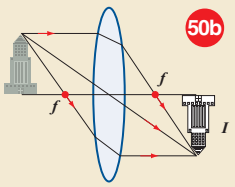
Lens and Mirror Properties

Parameters	+ sign	- sign
f focal length	converging lens concave mirror	diverging lens convex mirror
s object dist.	real object	virtual object
s' image dist.	real image	virtual image
h object size	erect	inverted
h' image size	erect	inverted

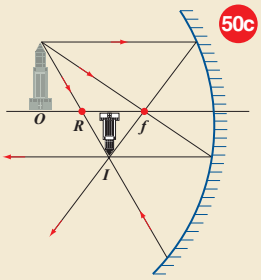
50a **Plane mirror: Law of Reflection**



Converging Lens 50b



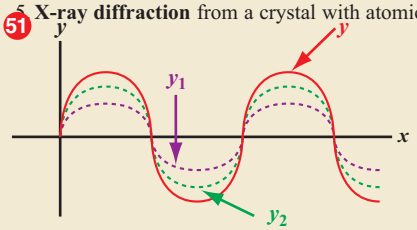
Spherical Concave Mirror 50c



C. Interference of Light Waves

1. **Goal**: Examine constructive and destructive interference of light waves
2. **Key Variables and Concepts**:
   a. **Constructive interference**: fig 51
   b. **Destructive interference**: fig 52
   c. **Huygens' Principle**: Each portion of wave front acts as a source of new waves
3. Diffraction of light, from a **grating** with spacing d, produces an interference pattern; $d\sin\theta = m\lambda$; (m = 0, 1, 2, 3,...)
4. **Single-slit experiment**, slit width a; destructive interference for $\sin\theta = \frac{m\lambda}{a}$; (m = 0, ±1, ±2...)
5. **X-ray diffraction** from a crystal with atomic spacing d; $2d\sin\theta = m\lambda$; (m = 0, 1, 2, 3,...)

51



52